N-vaton

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ABSTRACT

In general there are a large number of light scalar fields in the theories going beyond standard model, such as string theory, and some of them can be taken as the candidates of curvatons. For simplicity, we assume all of curvatons have the same decay rate and suddenly decay into radiation at the same time. In order to distinguish this scenario from the more general case, we call it "N-vaton". We use $\delta\mathcal{N}$ formalism to calculate the primordial power spectrum and bispectrum in N-vaton model and investigate various bounds on the non-Gaussianity parameter f_{NL}. A red tilted primordial power spectrum and a large value of f_{NL} can be naturally obtained if the curvature perturbation generated by inflaton also makes a significant contribution to the primordial power spectrum. As a realistic N-vaton model, we suppose that the axions in the KKLT compactifications of Type IIB string theory are taken as curvatons and a rich phenomenology is obtained.

1 Introduction

Most inflation models predict a nearly Gaussian distribution of the primordial curvature perturbation. Deviations from an exactly Gaussian distribution are characterized by a dimensionless parameter f_{NL} [1]. In the case of single field inflation model $f_{NL} \sim \mathcal{O}(n_s - 1)$ [2], which is constrained by WMAP ($n_s = 0.960^{+0.014}_{-0.013}$) [3] to be much less than unity. A Gaussian distribution of the primordial curvature perturbation is still consistent with WMAP five-year data [3]:

$$-9 < f_{NL}^{local} < 111 \quad \text{and} \quad -151 < f_{NL}^{equil} < 253 \quad (95\%\text{CL}), \tag{1.1}$$

where "local" and "equil" denote the shapes of the non-Gaussianity. In [4] the authors reported that a positive large non-Gaussianity

$$27 < f_{NL}^{local} < 147 \tag{1.2}$$

is detected at 95% C.L.. Planck is expected to bring the uncertainty of f_{NL}^{local} to be less than 5 [5]. If a large value of f_{NL} is confirmed by the forthcoming cosmological observations, the simplest model of inflation is ruled out and some very important new physics of the early Universe will be showed up.

In general a large number of light scalar fields are expected in the theories beyond the standard model, such as string theory. The consistent perturbative superstring theory can only live in ten-dimensional spacetime. To connect string theory with experiments, string theory must be compactified on some six-dimensional manifold and many dynamical moduli fields emerges in four dimensions. The typical number of moduli fields is $N \sim \mathcal{O}(10^2 - 10^3)$. One can expect that the expectation values of some of these scalar fields are displaced from the minimum of their potential due to the quantum fluctuations during inflation. Usually these scalar fields are subdominant during inflation and their fluctuations are initially of isocurvature type. After the end of inflation they are supposed to completely decay into thermalized radiation before primordial nucleosynthesis and thus the isocurvature perturbations generated by them are converted to be a final adiabatic perturbations. These scalar fields are called curvatons.

The curvaton mechanism to generate an initially adiabatic perturbation deep in the radiation era is proposed in [6–9]. The primordial curvature perturbation in curvaton model with single curvaton has been discussed in [6–13]. If many curvatons make contributions to the primordial density perturbation, the calculation becomes much more complicated [14–16]. Since curvaton model can give a large positive local-type non-Gaussianity, recently some topics related to curvaton model are discussed in [17–24].

In single-curvaton model f_{NL} is inverse proportional to the fraction of curvaton energy density in the energy budget at the epoch of curvaton decay. Smaller the energy density of curvaton, larger the non-Gaussianity. Since the curvaton mass is smaller than the Hubble parameter H_* during inflation, or equivalently its Compton wavelength is large compared to the curvature radius of the de Sitter space H_*^{-1}, the gravitational effects play a crucial role on the behavior of curvaton field in such a scenario. The typical energy density of curvaton field in such a background is roughly H_*^4, which leads to an upper bound on f_{NL} [17]: $f_{NL} < 522 \cdot r^{\frac{1}{4}}$ (up to an order one coefficient), where r is the tensor-scalar ratio.

Since multiplicity of scalar fields is generally expected, we focus on the multi-curvaton scenario in this paper. Here we consider a special case in which all of curvatons have the same decay rate and their masses are larger than the decay rate. In order to simplify the calculation of the primordial curvature perturbation, we assume all of curvatons suddenly decay into radiation at the same time. We give a name, "N-vaton", to this scenario. As a realistic N-vaton model, the axions in the KKLT compactification of Type IIB string theory are suggested to be curvatons. Based on the random matrix theory, the mass spectrum of axion obeys the Marcenko-Pastur law and a rich phenomenology of this model is shown up.

Our paper is organized as follows. In Sec. 2, we use the $\delta\mathcal{N}$ formalism [25–27] to calculate the primordial curvature perturbation on large scales in N-vaton model. The various bounds on f_{NL} are discussed in Sec. 3. In Sec. 4 we consider a more general case where the curvature perturbation generated by inflaton cannot be ignored and we find that the spectral index of primordial power spectrum can be red-tilted naturally. In Sec. 5, we propose a realistic N-vaton model in which the axions in the KKLT compactification of Type IIB string theory are taken as curvatons. At the end we give some discussions on N-vaton model in Sec. 6.

2 Primordial curvature perturbation

In this paper we consider that inflaton ϕ and curvatons σ_i are decoupled to each other. The action takes the form

$$S = \frac{M_p^2}{2} \int d^4x \sqrt{-g} R + \int d^4x \sqrt{-g} \left[\frac{1}{2}\dot{\phi}^2 + \sum_{i=1}^{N} \frac{1}{2}\dot{\sigma}_i^2 - V(\phi, \sigma_i) \right], \tag{2.1}$$

where $M_p = 2.438 \times 10^{18}$ GeV is the reduced Planck scale and the potential $V(\phi, \sigma_i)$ is given by

$$V(\phi, \sigma_i) = V(\phi) + \frac{1}{2} \sum_{i=1}^{N} m_i^2 \sigma_i^2. \tag{2.2}$$

During inflation the total energy density is dominated by inflaton potential $V(\phi)$ and the dynamics of the system is described by the equation of motion of inflaton and the Friedmann equation:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{dV(\phi)}{d\phi} = 0, \tag{2.3}$$

$$H^2 \equiv \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_p^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{2.4}$$

We also define some slow-roll parameters, such as

$$\epsilon = \frac{M_p^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2, \quad \eta = M_p^2\frac{V''(\phi)}{V(\phi)}. \tag{2.5}$$

If $\epsilon \ll 1$ and $|\eta| \ll 3$, inflaton slowly rolls down its potential.

In this paper we expand any field or perturbation at each order (n) as follows

$$\zeta(t,\mathbf{x}) = \zeta^{(1)}(t,\mathbf{x}) + \sum_{n=2}^{\infty}\frac{1}{n!}\zeta^{(n)}(t,\mathbf{x}). \tag{2.6}$$

We assume that the first-order term $\zeta^{(1)}$ is Gaussian and higher-order terms describe the non-Gaussianity of the full nonlinear ζ. Working in the framework of Fourier transformation of ζ, the primordial power spectrum $\mathcal{P}_\zeta$ is defined by

$$\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\rangle = (2\pi)^3\mathcal{P}_\zeta(k_1)\delta^3(\mathbf{k_1}+\mathbf{k_2}), \tag{2.7}$$

and the primordial bispectrum takes the form

$$\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = (2\pi)^3 B_\zeta(\mathbf{k_1},\mathbf{k_2})\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}). \tag{2.8}$$

The amplitude of the bispectrum relative to the power spectrum is parameterized by the non-Gaussianity parameter f_{NL}, i.e.

$$B_\zeta(\mathbf{k_1},\mathbf{k_2}) = \frac{6}{5}f_{NL}[\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + 2 \text{ perms}]. \tag{2.9}$$

The primordial density perturbation can be described in terms of the nonlinear curvature perturbation on uniform density hypersurfaces [28]

$$\zeta(t,\mathbf{x}) = \delta\mathcal{N}(t,\mathbf{x}) + \frac{1}{3}\int_{\bar{\rho}(t)}^{\rho(t,\mathbf{x})}\frac{d\tilde{\rho}}{\tilde{\rho}+\tilde{p}}, \tag{2.10}$$

where $\mathcal{N} = \int H dt$ is the integrated local expansion, $\bar{\rho}$ is the homogeneous density in the background model, $\tilde{\rho}$ is the local density and $\tilde{p}$ is the local pressure.

After inflation inflaton decays into radiations which dominate the total energy density of our universe. In the radiation dominated era the Hubble parameter H goes like $\sim a^{-2}$. Once the Hubble parameter drops below the mass of curvaton field the field starts to oscillate. Nonlinear evolution of the values of curvatons on large scale is possible if the potential of curvatons deviates from a purely quadratic potential away from their minimums [29, 30]. Thus, in general, the initial amplitude of curvaton oscillations $\sigma_{i,o}$ is some function of the field value $\sigma_{i,*}$ at the Hubble exit [1]:

$$\sigma_{i,o} = g_i(\sigma_{i,*}). \tag{2.11}$$

When the curvaton starts to oscillate about the minimum of its potential, but before it decays, it behaves like pressureless dust ($\rho_{\sigma_{i,o}} \sim a^{-3}$) and the nonlinear curvature perturbation on uniform-curvaton density hypersurfaces is given by

$$\zeta_{\sigma_{i,o}}(t, \mathbf{x}) = \delta\mathcal{N}(t, \mathbf{x}) + \int_{\bar{\rho}_{\sigma_{i,o}}(t)}^{\rho_{\sigma_{i,o}}(t,\mathbf{x})} \frac{d\tilde{\rho}_{\sigma_{i,o}}}{3\tilde{\rho}_{\sigma_{i,o}}}. \tag{2.12}$$

The curvaton density on spatially flat hypersurfaces is

$$\rho_{\sigma_{i,o}}|_{\delta\mathcal{N}=0} = e^{3\zeta_{\sigma_{i,o}}}\bar{\rho}_{\sigma_{i,o}}. \tag{2.13}$$

The quantum fluctuations in a weakly coupled field, such as curvaton, at Hubble exit during inflation are expected to be well described by a Gaussian random field [31]. So we have

$$\sigma_{i,*} = \bar{\sigma}_{i,*} + \delta\sigma_{i,*}, \tag{2.14}$$

without higher-order nonlinear terms. During the curvaton oscillation we expand the energy density $\rho_{\sigma_{i,o}} = \frac{1}{2}m_i^2\sigma_{i,o}^2$ and $\zeta_{\sigma_{i,o}}$ to second order:

$$\rho_{\sigma_{i,o}} = \bar{\rho}_{\sigma_{i,o}}\left[1 + 2X_i + (1 + h_i)X_i^2\right], \tag{2.15}$$

$$\zeta_{\sigma_{i,o}} = \zeta^{(1)}_{\sigma_{i,o}} + \frac{1}{2}\zeta^{(2)}_{\sigma_{i,o}}, \tag{2.16}$$

where $\bar{\rho}_{\sigma_{i,o}} = \frac{1}{2}m_i^2\bar{\sigma}_{i,o}^2$, $\bar{\sigma}_{i,o} \equiv g_i(\sigma_{i,*})$, and

$$X_i = \frac{\delta\sigma^{(1)}_{i,o}}{\bar{\sigma}_{i,o}}, \tag{2.17}$$

$$h_i = \frac{g_i g_i''}{g_i'^2}. \tag{2.18}$$

[1]In this paper the subscript $*$ denotes the quantity evaluated at the Hubble exit.

Here prime denotes the derivative with respect to $\sigma_{i,*}$. Order by order, from Eq.(2.13) we have

$$\zeta^{(1)}_{\sigma_{i,o}} = \frac{2}{3} X_i, \tag{2.19}$$

$$\zeta^{(2)}_{\sigma_{i,o}} = -\frac{3}{2}\left(1-h_i\right)\left(\zeta^{(1)}_{\sigma_{i,o}}\right)^2. \tag{2.20}$$

In N-vaton model, we assume that the curvatons have the same decay rate Γ_σ. When the Hubble parameter drops below Γ_σ, all of curvatons decay into radiations. In order to get analytic expressions, we work in the sudden-decay approximation which means that all of curvatons suddenly decay into radiations at the time t_D when $H = \Gamma_\sigma$. For simplicity, we assume $m_i > \Gamma_\sigma$ for $i = 1, 2, ..., N$ and then all of curvatons begin oscillating before they decay.

The curvatons-decay hypersurface is a uniform-density hypersurface and thus from Eq.(2.10) the perturbed expansion on this hypersurface is $\delta\mathcal{N} = \zeta$, where ζ is the total curvature perturbation at curvatons-decay hypersurface. Before the curvatons decay, there have been radiations produced by decay of inflaton. Since the equation of state of radiation is $p_r = \rho_r/3$, the curvature perturbation related to radiations is

$$\zeta_r = \zeta + \frac{1}{4}\ln\frac{\rho_r}{\bar{\rho}_r}. \tag{2.21}$$

The curvatons behave like pressureless dust ($p_{\sigma_i} = 0$) and thus

$$\zeta_{\sigma_{i,o}} = \zeta + \frac{1}{3}\ln\frac{\rho_{\sigma_{i,o}}}{\bar{\rho}_{\sigma_{i,o}}}. \tag{2.22}$$

In the absence of interations, the curvature perturbations ζ_r and $\zeta_{\sigma_{i,o}}$ are conserved respectively and the above two equations can be written as

$$\rho_r = \bar{\rho}_r e^{4(\zeta_r-\zeta)}, \tag{2.23}$$

$$\rho_{\sigma_{i,o}} = \bar{\rho}_{\sigma_{i,o}} e^{3(\zeta_{\sigma_{i,o}}-\zeta)}. \tag{2.24}$$

At the time of curvatons decay, the total energy density ρ_{tot} is conserved, i.e.

$$\rho_r(t_D, \mathbf{x}) + \sum_{i=1}^{N}\rho_{\sigma_{i,o}}(t_D, \mathbf{x}) = \bar{\rho}_{tot}(t_D). \tag{2.25}$$

Requiring that the total energy density is uniform on the decay surface, we have

$$\left(1-\Omega_{\sigma,D}\right) e^{4(\zeta_r-\zeta)} + \sum_{i=1}^{N}\Omega_{\sigma_i,D} e^{3(\zeta_{\sigma_{i,o}}-\zeta)} = 1, \tag{2.26}$$

where $\Omega_{\sigma_i,D} = \bar{\rho}_{\sigma_i,D}/\bar{\rho}_{tot}$ is the fraction of curvaton energy density in the energy budget at the time of curvaton decay, and

$$\Omega_{\sigma,D} \equiv \sum_{i=1}^{N} \Omega_{\sigma_i,D}. \tag{2.27}$$

Actually ζ_r is generated by the fluctuation of inflaton ϕ during inflation, namely $\zeta_r = \zeta_\phi$. In N-vaton scenario, usually we assume the curvature perturbation caused by inflaton is relatively small and can be neglected. The more general case with $\zeta_r = \zeta_\phi \neq 0$ is discussed in Appendix A. Here we consider $\zeta_r = 0$ and Eq.(2.26) gives

$$e^{4\zeta} - \left(\sum_{i=1}^{N} \Omega_{\sigma_i,D} e^{3\zeta_{\sigma_{i,o}}}\right) e^{\zeta} + \Omega_{\sigma,D} - 1 = 0. \tag{2.28}$$

Order by order, from Eq.(2.28) we have

$$\zeta^{(1)} = A \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta^{(1)}_{\sigma_{i,o}}, \tag{2.29}$$

and

$$\zeta^{(2)} = \frac{1}{4 - \Omega_{\sigma,D}} \left[\frac{9}{2} \sum_{i=1}^{N} \Omega_{\sigma_i,D}(1 + h_i) \left(\zeta^{(1)}_{\sigma_{i,o}}\right)^2 - (8 + \Omega_{\sigma,D}) \left(\zeta^{(1)}\right)^2\right], \tag{2.30}$$

where

$$A = \frac{3}{4 - \Omega_{\sigma,D}}. \tag{2.31}$$

The total curvature perturbation up to second order is

$$\begin{aligned} \zeta &= \zeta^{(1)} + \frac{1}{2}\zeta^{(2)} = A \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta^{(1)}_{\sigma_{i,o}} \\ &+ \frac{3A}{4} \sum_{i=1}^{N} \Omega_{\sigma_i,D}(1 + h_i) \left(\zeta^{(1)}_{\sigma_{i,o}}\right)^2 - (1 + \frac{A}{2}\Omega_{\sigma,D}) A^2 \left(\sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta^{(1)}_{\sigma_{i,o}}\right)^2. \end{aligned} \tag{2.32}$$

Assume that the two different curvatons are uncorrelated with each other and then

$$\langle \zeta^{(1)}_{\sigma_{i,o}}(\mathbf{k_1}) \zeta^{(1)}_{\sigma_{j,o}}(\mathbf{k_2})\rangle = (2\pi)^3 \mathcal{P}_{\zeta_{\sigma_{i,o}}}(k_1) \delta_{ij} \delta^3(\mathbf{k_1} + \mathbf{k_2}). \tag{2.33}$$

Using Eq.(2.29), we can easily calculate the primordial power spectrum:

$$\mathcal{P}_\zeta = A^2 \sum_{i=1}^{N} \Omega^2_{\sigma_i,D} \mathcal{P}_{\zeta_{\sigma_{i,o}}}. \tag{2.34}$$

For convenience, we introduce a new parameter α_i as follows

$$\mathcal{P}_{\zeta_{\sigma_{i,o}}} = A^{-2} \alpha_i \mathcal{P}_\zeta. \tag{2.35}$$

The constraint on the coefficients α_i is

$$\sum_{i=1}^{N}\Omega^2_{\sigma_i,D}\alpha_i = 1. \tag{2.36}$$

Similarly we can also calculate the primordial bispectrum:

$$\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = &\frac{3A^3}{4}\sum_{i,j,k=1}^{N}\Omega_{\sigma_i,D}\Omega_{\sigma_j,D}\Omega_{\sigma_k,D}\langle\zeta^{(1)}_{\sigma_{i,o}}(\mathbf{k_1})\zeta^{(1)}_{\sigma_{j,o}}(\mathbf{k_2})(\zeta^{(1)}_{\sigma_{k,o}} * \zeta^{(1)}_{\sigma_{k,o}})(\mathbf{k_3})\rangle \\ &- (1+\frac{A}{2}\Omega_{\sigma,D})A^4\sum_{i,j,k,l=1}^{N}\Omega_{\sigma_i,D}\Omega_{\sigma_j,D}\Omega_{\sigma_k,D}\Omega_{\sigma_l,D} \\ &\times \langle\zeta^{(1)}_{\sigma_{i,o}}(\mathbf{k_1})\zeta^{(1)}_{\sigma_{j,o}}(\mathbf{k_2})(\zeta^{(1)}_{\sigma_{k,o}} * \zeta^{(1)}_{\sigma_{l,o}})(\mathbf{k_3})\rangle \\ &+ 2 \text{ permutations of } \{\mathbf{k_1},\mathbf{k_2},\mathbf{k_3}\},\end{aligned} \tag{2.37}$$

where $*$ denotes a convolution as follows

$$(\zeta^{(1)}_{\sigma_{i,o}} * \zeta^{(1)}_{\sigma_{j,o}})(\mathbf{k}) = \frac{1}{(2\pi)^3}\int d^3\mathbf{q}\zeta^{(1)}_{\sigma_{i,o}}(\mathbf{q})\zeta^{(1)}_{\sigma_{j,o}}(\mathbf{k}-\mathbf{q}). \tag{2.38}$$

After straightforward calculations, we get

$$\begin{aligned}\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\zeta(\mathbf{k_3})\rangle = &\left[\frac{3}{2A}\sum_{i=1}^{N}\Omega^3_{\sigma_i,D}\alpha_i^2(1+h_i) - (2+A\Omega_{\sigma,D})\right] \\ &\times (2\pi)^3[\mathcal{P}_\zeta(k_1)\mathcal{P}_\zeta(k_2) + 2\ perms]\delta^3(\mathbf{k_1}+\mathbf{k_2}+\mathbf{k_3}).\end{aligned} \tag{2.39}$$

Using the definition of f_{NL} in Eq.(2.9), we have

$$f_{NL} = \frac{5}{4A}\sum_{i=1}^{N}\Omega^3_{\sigma_i,D}\alpha_i^2(1+h_i) - (\frac{5}{3}+\frac{5A}{6}\Omega_{\sigma,D}). \tag{2.40}$$

For single curvaton, the solution of Eq.(2.36) is $\alpha = 1/\Omega^2_{\sigma,D}$ and then $f_{NL}^{single} = \frac{5}{4f_D}(1+h) - \frac{5}{3} - \frac{5f_D}{6}$, where $f_D = A\Omega_{\sigma,D}$. It is the same as the result in the literatures.

To compare with the cosmological observations, we introduce a "dimensionless" power spectrum P_ζ which is defined by

$$\langle\zeta(\mathbf{k_1})\zeta(\mathbf{k_2})\rangle \equiv \frac{2\pi^2}{k_1^3}P_\zeta\delta^3(\mathbf{k_1}+\mathbf{k_2}). \tag{2.41}$$

The power spectrum of $\delta_{\sigma_{i,*}}$ is given by

$$P_{\delta_{\sigma_{i,*}}} = \left(\frac{H_*}{2\pi}\right)^2. \tag{2.42}$$

According to Eq.(2.17) and (2.19), we have

$$P_{\zeta_{\sigma_{i,o}}} = \frac{4}{9}q_i^2 P_{\delta_{\sigma_{i,*}}} = \frac{1}{9\pi^2}q_i^2H_*^2, \tag{2.43}$$

where

$$q_i = g_i'/g_i. \tag{2.44}$$

The value of α_i takes the form

$$\alpha_i = A^2 P_{\zeta_{\sigma_{i,o}}}/P_\zeta = \frac{A^2}{9\pi^2}\frac{q_i^2 H_*^2}{P_\zeta}. \tag{2.45}$$

Based on Eq.(2.34), the amplitude of the primordial power spectrum P_ζ becomes

$$P_\zeta = \frac{A^2}{9\pi^2}\sum_{i=1}^{N}\Omega_{\sigma_i,D}^2 q_i^2 H_*^2. \tag{2.46}$$

In [3] WMAP normalization of the primordial power spectrum is

$$P_{\zeta,WMAP} = 2.457^{+0.092}_{-0.093}\times 10^{-9}. \tag{2.47}$$

On the other hand, the amplitude of primordial power spectrum generated by inflaton is

$$P_{\zeta_\phi} = \frac{H_*^2/M_p^2}{8\pi^2\epsilon}, \tag{2.48}$$

which should be much smaller than $P_{\zeta,WMAP}$, namely

$$H_* \ll 4.4\times 10^{-4}\sqrt{\epsilon}M_p. \tag{2.49}$$

Gravitational wave perturbation (tensor perturbation) is also generated during inflation in N-vaton model. The tensor perturbation only depends on the inflation scale and its amplitude is given by

$$P_T = \frac{H_*^2/M_p^2}{\pi^2/2}. \tag{2.50}$$

Usually we define a new parameter named the tensor-scalar ratio r to measure the amplitude of the tensor perturbations:

$$r = P_T/P_\zeta. \tag{2.51}$$

So the Hubble parameter during inflation is related to the tensor-scalar ratio by

$$H_* = \frac{\pi}{\sqrt{2}}P_\zeta^{1/2} r^{\frac{1}{2}} M_p. \tag{2.52}$$

Using WMAP normalization (2.47), we get $H_* = 10^{-4} r^{\frac{1}{2}} M_p$. If the density perturbation is dominated by inflaton fluctuation, we have $r = 16\epsilon$. In curvaton/N-vaton scenario, the density perturbation caused by inflaton is subdominant, and thus the inflation scale should be relatively low, i.e. $r < 16\epsilon$.

Note: Going beyond the sudden-decay approximation, the single-curvaton model was studied in [10, 13] in detail. We hope that one can do it for N-vaton model in the future. In this paper we always adopt the sudden-decay approximation.

3 Bound on the non-Gaussianity parameter f_{NL}

In this section we consider the case in which the Hubble parameter is roughly a constant during inflation. If $\epsilon = -\dot{H}/H^2$ is large, the variation of inflaton is larger than the Planck scale [32]. Usually this kind of inflation model cannot be embedded into string theory [33–38]. For simplicity we also assume the values of curvatons don't evolve between Hubble exit during inflation and the beginning of their oscillations. So we have $\sigma_{i,o} = g_i(\sigma_{i,*}) = \sigma_{i,*}$, and thus $q_i = 1/\sigma_{i,*}$ and $h_i = 0$. Here we are interested in the case of large non-Gaussianity. From Eq.(2.40), a large non-Gaussianity can be obtained if $\alpha_i \gg 1$, but $\Omega_{\sigma,D}$ is not necessarily required to be much smaller than 1. For example, if one or more coefficients α_i are large enough and $\Omega^2_{\sigma_i,D}\alpha_i$ takes a finite value for $\Omega_{\sigma_i,D} \ll 1$, f_{NL} can be large even when $\Omega_{\sigma,D} = 1$ because of $f_{NL} \sim (\Omega^2_{\sigma_i,D}\alpha_i)^2/\Omega_{\sigma_i,D}$. This case can be possibly achieved only for multiple curvatons. In the single-curvaton model, $\Omega^2_{\sigma,D}\alpha = 1$ and $f_{NL} \sim 1/\Omega_{\sigma,D}$. However whether the above conditions can be naturally realized in a concrete N-vaton model is still an open question and we will return to this problem in some future work. Here we only give a brief discussion for the case with two curvatons in the Appendix B.

From now on, we only focus on the case with $\Omega_{\sigma,D} \ll 1$ for simplicity. In this case a large non-Gaussianity is also expected. Now $A = 3/4$, the amplitude of primordial power spectrum and the non-Gaussianity parameter in Eq.(2.46) and (2.40) are respectively simplified to be

$$P_\zeta = \frac{1}{16\pi^2}\sum_{i=1}^{N}\Omega^2_{\sigma_i,D}\frac{H_*^2}{\sigma_{i,*}^2}, \tag{3.1}$$

and

$$f_{NL} = \frac{5}{3}\sum_{i=1}^{N}\Omega^3_{\sigma_i,D}\alpha_i^2. \tag{3.2}$$

Since α_i is only constrained by Eq.(2.36), usually we need more information if we want to constrain the non-Gaussianity parameter f_{NL}.

3.1 Lower bound on f_{NL}

Let's introduce a very useful inequality

$$\sum_{i=1}^{N}u_i^2\cdot\sum_{j=1}^{N}v_i^2 \geq \left(\sum_{i=1}^{N}u_iv_i\right)^2, \tag{3.3}$$

where $u_i \geq 0$ and $v_i \geq 0$ for $i = 1, 2, ..., N$. The equality in Eq.(3.3) is satisfied only when $u_i/u_j = v_i/v_j$ for $i, j = 1, 2, ..., N$. Using this inequality, we immediately find

$$\sum_{i=1}^{N} \Omega_{\sigma_i,D} \sum_{j=1}^{N} \Omega^3_{\sigma_j,D} \alpha_j^2 \geq \left(\sum_{i=1}^{N} \Omega^2_{\sigma_i,D} \alpha_i \right)^2 . \tag{3.4}$$

Taking Eq.(2.36) into account, we find the non-Gaussianity parameter f_{NL} in Eq.(3.2) is bounded from below, namely

$$f_{NL} \geq \frac{5}{3\Omega_{\sigma,D}}. \tag{3.5}$$

The equality is satisfied when $\alpha_i \Omega_{\sigma_i,D} = \theta$ which is a constant. We can easily check it. In this special case the solution of Eq.(2.36) is given by

$$\theta = 1/\Omega_{\sigma,D}, \tag{3.6}$$

and then

$$\alpha_i = \frac{1}{\Omega_{\sigma_i,D} \Omega_{\sigma,D}}. \tag{3.7}$$

Instituting this solution into Eq.(3.2), we get

$$f_{NL} = \frac{5}{3\Omega_{\sigma,D}}. \tag{3.8}$$

Keeping $\Omega_{\sigma,D}$ fixed, the non-Gaussianity parameter f_{NL} in N-vaton model is not less than that in single-curvaton model.

3.2 Upper bound on f_{NL}

In this subsection we take more information into account. Because we only focus on the limit of $\Omega_{\sigma,D} \ll 1$, the radiation produced by inflaton is always dominant before the curvaton decay. After that curvatons oscillate around their minimums $\sigma_i = 0$ and their energy density decreases as a^{-3}. Once the Hubble parameter drops below Γ_σ, the curvatons energy is converted into radiations. Similar to the arguments in [8, 10], the energy density parameter $\Omega_{\sigma_i,D}$ at the time of curvatons decay is given by

$$\Omega_{\sigma_i,D} = \frac{\sigma^2_{i,*}}{6M_p^2} \left(\frac{m_i}{\Gamma_\sigma} \right)^{\frac{1}{2}} . \tag{3.9}$$

Instituting the above equation into Eq.(3.1), the amplitude of primordial power spectrum becomes

$$P_\zeta = \frac{H_*^2}{(24\pi)^2 M_p^4 \Gamma_\sigma} \sum_{i=1}^{N} m_i \sigma^2_{i,*}, \tag{3.10}$$

or equivalently,

$$\sum_{i=1}^{N} m_i \sigma_{i,*}^2 = (24\pi)^2 P_\zeta \frac{M_p^4 \Gamma_\sigma}{H_*^2}. \tag{3.11}$$

The WMAP normalization gives a constraint on $\sum_{i=1}^{N} m_i \sigma_{i,*}^2$.

In this section $A = 3/4$, $g_i(\sigma_{i,*}) = \sigma_{i,*}$ and $q_i = 1/\sigma_{i,*}$. Eq.(2.45) is simplified to be

$$\alpha_i = \frac{1}{16\pi^2} \frac{H_*^2/\sigma_{i,*}^2}{P_\zeta}, \tag{3.12}$$

and then

$$\alpha_i \Omega_{\sigma_{i,D}} = \frac{r}{192}\sqrt{\frac{m_i}{\Gamma_\sigma}}. \tag{3.13}$$

If $m_i = m$ for $i = 1, 2, ..., N$, $\alpha_i \Omega_{\sigma_{i,D}}$ is a constant and the inequality in Eq.(3.5) is saturated. Now we have

$$\theta = \frac{r}{192}\sqrt{\frac{m}{\Gamma_\sigma}}, \tag{3.14}$$

and

$$f_{NL} = \frac{5}{576} r \sqrt{\frac{m}{\Gamma_\sigma}}. \tag{3.15}$$

In general different curvatons σ_i has different mass m_i. Using Eq.(3.2), (3.9) and (3.12), we find the non-Gaussianity parameter f_{NL} takes the form

$$f_{NL} = 3 \times 10^{-7} P_\zeta^{-2} \frac{H_*^4}{M_p^6 \Gamma_\sigma^{3/2}} \sum_{i=1}^{N} m_i^{\frac{3}{2}} \sigma_{i,*}^2. \tag{3.16}$$

When $m_i = m$ for $i = 1, 2, ..., N$, we can easily check that this results is the same as (3.15).

How to determine the value of $\sigma_{i,*}$ is a crucial problem in curvaton/N-vaton model. In the literatures $\sigma_{i,*}$ are taken as free parameters. In classical level it is correct. However for a scalar field χ in de Sitter space, if its mass is much smaller than H_*, its Compton wavelength is large compared to the curvature radius of the background H_*^{-1} and the gravitational effects may play a crucial role on its behavior. In [39–41] the authors explicitly showed that the quantum fluctuation of a light scalar field χ with mass m_χ in de Sitter space gives it a non-zero expectation value of χ^2

$$\langle \chi^2 \rangle = \frac{3H_*^4}{8\pi^2 m_\chi^2}. \tag{3.17}$$

This result is reliable for a light scaler field with $m_\chi \ll \sqrt{2} H_*$ in a long-lived, quasi-de Sitter inflation. Here we also ignore the possible corrections from the cubic, or higher-power terms in the curvaton potential. So the typical or average energy density of the

scalar field χ is $\frac{3H_*^4}{16\pi^2}$. Since the masses of curvatons are assumed to be much smaller than H_*, the total energy density of curvatons can be estimated as $\frac{3NH_*^4}{16\pi^2}$, which implies

$$\sum_{i=1}^{N} m_i^2 \sigma_{i,*}^2 = \frac{3NH_*^4}{8\pi^2}. \tag{3.18}$$

Using the inequality (3.3), Eq.(3.11) and (3.18), we have

$$\sum_{i=1}^{N} m_i^{\frac{3}{2}} \sigma_{i,*}^2 \leq 6\sqrt{6}(NP_\zeta \Gamma_\sigma)^{\frac{1}{2}} H_* M_p^2. \tag{3.19}$$

We see that the non-Gaussianity parameter f_{NL} in Eq.(3.16) is bounded from above:

$$f_{NL} \leq 4.41 \times 10^{-6} P_\zeta^{-\frac{3}{2}} N^{\frac{1}{2}} \frac{H_*^5}{M_p^4 \Gamma_\sigma}. \tag{3.20}$$

The inequality (3.20) is saturated when these curvatons fields have the same mass: $m_1 = m_2 = ... = m_N = m$. Now $\alpha_i \Omega_{\sigma_i,D}$ is a constant and the lower bound (3.5) is also saturated. One point we want to stress is that $\Omega_{\sigma,D}$ is not kept fixed. Keeping the inflation scale H_* (or tensor-scalar ratio r), the number of the curvatons N and the curvaton decay rate Γ_σ fixed, the non-Gaussianity f_{NL} is maximized in the case where all of the curvatons have the same mass. We discuss this special case in Sec. 3.4 in detail.

3.3 Adiabatic condition

In [21] the author pointed out that the curvaton model is free from the constraint of isocurvature perturbation in WMAP [3] if the cold dark matter (CDM) is not the direct decay product of the curvatons and CDM is generated after the curvatons decay completely. So does N-vaton. Denote H_{cdm} as the Hubble parameter when CDM is generated and thus $H_{cdm} < \Gamma_\sigma$. The Hubble parameter H_{cdm} is related to the temperature T_{cdm} at the epoch of CDM creation by $H_{cdm} = T_{cdm}^2/M_p$. Therefore

$$\Gamma_\sigma > \frac{T_{cdm}^2}{M_p}. \tag{3.21}$$

Combining with Eq.(3.20), we find

$$T_{cdm} < 1.87 \times 10^{12} N^{\frac{1}{4}} r^{\frac{5}{4}} f_{NL}^{-\frac{1}{2}} \text{ GeV}, \tag{3.22}$$

where we use Eq.(2.52) and WMAP normalization $P_\zeta = P_{\zeta,WMAP}$. In [42] the relationship between T_{cdm} and the mass of CDM M_{cdm} is roughly given by

$$M_{cdm} \simeq 20 T_{cdm}. \tag{3.23}$$

So the mass of CDM is bounded from above

$$M_{cdm} < 3.7 \times 10^{13} N^{\frac{1}{4}} r^{\frac{5}{4}} f_{NL}^{-\frac{1}{2}} \text{ GeV}. \tag{3.24}$$

For example, $N \sim 10^3$, $r \sim 10^{-4}$ and $f_{NL} \sim 50$, the mass of CDM is less than 3×10^8 GeV. On the other hand, f_{NL} is bounded by $1/M_{cdm}^2$ from above.

3.4 The case with $m_i = m$ for $i = 1, 2, ..., N$

In this case the constraint coming from the amplitude of power spectrum (3.11) and the estimation of the total energy density of curvaton during inflation (3.18) are respectively simplified to be

$$\sigma_T^2 \equiv \sum_{i=1}^{N} \sigma_{i,*}^2 = (24\pi)^2 P_\zeta \frac{M_p^4 \Gamma_\sigma}{H_*^2 m}, \tag{3.25}$$

and

$$\sigma_T^2 = \frac{3NH_*^4}{8\pi^2 m^2}. \tag{3.26}$$

According to the above two equations, we find that curvaton mass m is related to curvaton decay rate Γ_σ by

$$m = 6.68 \times 10^{-6} P_\zeta^{-1} N \frac{H_*^6}{M_p^4 \Gamma_\sigma}. \tag{3.27}$$

Keeping Γ_σ fixed, the mass of curvatons m in N-vaton is N times of that in single-curvaton model.

In Sec. 3.2, we argue that the upper bound on the non-Gaussianity parameter f_{NL} in Eq.(3.20) is saturated when the curvatons have the same mass and now we have

$$f_{NL} = 4.41 \times 10^{-6} P_\zeta^{-\frac{3}{2}} N^{\frac{1}{2}} \frac{H_*^5}{M_p^4 \Gamma_\sigma}. \tag{3.28}$$

Keeping Γ_σ and H_* fixed, larger the number of curvatons, larger the non-Gaussianity parameter f_{NL}. On the other hand, if N is fixed, smaller Γ_σ, larger f_{NL}. However, similar to the argument in [29], the curvaton decay rate is larger than the gravitational strength decay rate, i.e.

$$\Gamma_\sigma > \frac{1}{c^4} \frac{m^3}{M_p^2}, \tag{3.29}$$

where c is supposed to be an order one coefficient which we have not known exactly. The curvaton decay rate cannot be arbitrary small. Substituting Eq.(3.27) into (3.29), we find that Γ_σ is bounded from below by the number of curvatons

$$\Gamma_\sigma > 1.3 \times 10^{-4} c^{-1} P_\zeta^{-\frac{3}{4}} N^{\frac{3}{4}} \frac{H_*^{9/2}}{M_p^{7/2}}. \tag{3.30}$$

The lower bound on the curvatons decay rate rises as the number of curvatons increases. Combing with Eq.(3.28), we find f_{NL} is bounded by the tensor-scalar ratio from above

$$f_{NL} < 0.034 \cdot c \cdot P_\zeta^{-\frac{3}{4}} N^{-\frac{1}{4}} \frac{H_*^{1/2}}{M_p^{1/2}} = 10^3 \cdot c \cdot \left(\frac{r}{N}\right)^{\frac{1}{4}}. \tag{3.31}$$

For $N = 1$, our result is the same as that in [17] where we go beyond sudden-decay approximation and ignore the coefficient $3/8\pi^2$ when we estimated the expectation value of square of the curvaton field. Here we introduce an order one coefficient c to encode the uncertain coefficient in the calculations. On the other hand, using Eq.(3.27), (3.28) and (3.31), we obtain

$$f_{NL} < \frac{c^{2/3}}{P_\zeta^{2/3}} \left(\frac{m/N}{1.3 \times 10^3 M_p}\right)^{\frac{1}{3}} = c^{\frac{2}{3}} \left(\frac{m/N}{2 \times 10^4 \text{ GeV}}\right)^{\frac{1}{3}}. \tag{3.32}$$

Requiring the mass of each curvatons is smaller than the Hubble parameter H_* leads to another bound on f_{NL}, i.e.

$$f_{NL} < 2.3 \times 10^3 \cdot c^{\frac{2}{3}} \cdot \frac{r^{1/6}}{N^{1/3}}. \tag{3.33}$$

If $r > 2 \times 10^4/(c^4 N)$, the constraint in Eq.(3.33) is more restricted than that in (3.31). To summarize, the bound on the non-Gaussianity parameter f_{NL} is given by

$$f_{NL} < \min\left[10^3 \cdot c \cdot \left(\frac{r}{N}\right)^{\frac{1}{4}}, \quad 2.3 \times 10^3 \cdot c^{\frac{2}{3}} \cdot \frac{r^{1/6}}{N^{1/3}}\right]. \tag{3.34}$$

We see the the constraint on f_{NL} in N-vaton model is more stringent than that in single-curvaton model. The reason is that the energy density of each curvaton during inflation is roughly H_*^4 and thus the total energy density of curvatons in N-vaton model is much larger than single curvaton energy density. Larger the number of curvatons, larger $\Omega_{\sigma,D}$. Since $f_{NL} \sim 1/\Omega_{\sigma,D}$, the non-Gaussianity is suppressed in N-vaton model by the number of curvatons N. A reasonable estimation of the number of curvatons in string theory might be 10^3 and $r \leq 10^{-3}$ if we require the variation of inflaton be smaller than Planck scale [34]. If so, $f_{NL} \leq 32 \cdot c$. Usually f_{NL} in N-vaton model should be less than 10^2. For $f_{NL} > 10$, $r > 10^{-8} N$ and $m > 10^7 N$ GeV. Typically we have $N \sim 10^3$ and then $m > 10^{10}$ GeV, $r > 10^{-5}$ which implies $H_* > 10^{12}$ GeV.

3.5 N-vaton vs. single-curvaton model

According to previous discussions, the non-Gaussianity parameter f_{NL} in N-vaton model is larger than that in single-curvaton model if the decay rate of curvaton is kept fixed,

and the maximum value of f_{NL} in N-vaton is obtained when all of curvatons have the same mass. The maximum value of f_{NL} is $\sqrt{N}$ times of that in single-curvaton model. However now the curvaton mass in N-vaton is N times of that in single-curvaton model. The requirement that the curvaton decay rate be larger than the gravitational strength decay rate in N-vaton model becomes much more stringent than that in single-curvaton model. That is why the upper bound on f_{NL} in Eq.(3.31) is suppressed by a factor $1/N^{\frac{1}{4}}$.

On the other hand, we consider the mass of different curvaton is quite different from each other. For simplicity, we estimate $\sigma_{i,*} \sim H_*^2/m_i$. According to Eq.(3.10) and (3.16), the contributions to the amplitude of primordial power spectrum and non-Gaussianity parameter from curvaton σ_i are respectively $P_{\zeta_i} \sim H_*^6/(M_p^4 \Gamma_\sigma m_i)$ and $f_{NL,i} \sim H_*^8/(P_\zeta^2 M_p^6 \Gamma_\sigma^{\frac{3}{2}} m_i^{\frac{1}{2}})$. If the lightest curvaton σ_L is much lighter than other curvatons, the total primordial power spectrum and non-Gaussianity are roughly contributed by σ_L. Now N-vaton model is reduced to single-curvaton model and $f_{NL} \sim m_L/(P_\zeta^{\frac{1}{2}} H_*)$. Similarly, requiring $\Gamma_\sigma > m_L^3/(c^4 M_p^2)$ yields $f_{NL} < 10^3 \cdot c \cdot r^{\frac{1}{4}}$.

4 Spectral index and non-Gaussianity

The spectral index of the primordial power spectrum generated by curvatons is defined as

$$n_s^{nc} \equiv 1 + \frac{d \ln P_\zeta^{nc}}{d \ln k} = 1 - 2\epsilon + 2\eta_{\sigma\sigma}, \tag{4.1}$$

where

$$\eta_{\sigma\sigma} \equiv \sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 \alpha_i \frac{1}{3H_*^2} \frac{d^2 V(\sigma_i)}{d\sigma_i^2} = \sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 \alpha_i \frac{m_i^2}{3H_*^2}. \tag{4.2}$$

If the primordial power spectrum is dominated by the curvature perturbation generated by curvatons, $n_s = n_s^{nc}$. The masses of curvatons are assumed to be much smaller than H_* and then $n_s \simeq 1-2\epsilon$. For $n_s = 0.96$, $\epsilon = 0.02$ which might be realized in landscape inflation [43–45] or the monodromies [46]. However in this case the Hubble parameter H during inflation cannot be taken as a constant any more. In [24], we showed that the values of the curvatons depend on the initial condition of inflation which should be fine-tuned to achieve the suitable amplitude of primordial power spectrum and non-Gaussianity parameter f_{NL}. It is quite unnatural. Usually a closely scale-invariant curvature perturbation generated by curvaton is expected in curvaton/N-vaton scenario.

On the other hand, the spectral index of the primordial power spectrum generated by inflaton is

$$n_s^{inf} \equiv 1 + \frac{d \ln P_\zeta^{inf}}{d \ln k} = 1 - 6\epsilon + 2\eta. \tag{4.3}$$

In some inflation models, $\epsilon \simeq 0$, but the order of magnitude of η can be $-\mathcal{O}(10^{-1})$ to $-\mathcal{O}(10^{-2})$. If the inflaton fluctuation makes a significant contribution to the total primordial power spectrum, a red-tilted primordial power spectrum is possibly obtained. We calculate the curvature perturbation for this scenario in Appendix A. Introduce a parameter β to measure the relative amplitude of power spectrum generated by curvatons:

$$\beta = P_\zeta^{nc}/P_\zeta^{tot}, \tag{4.4}$$

and then

$$P_\zeta^{inf} = (1-\beta)P_\zeta^{tot}. \tag{4.5}$$

Now the spectral index becomes

$$\begin{aligned} n_s \equiv 1 + \frac{d\ln P_\zeta^{tot}}{d\ln k} &= \beta n_s^{nc} + (1-\beta)n_s^{inf} \\ &= 1-(6-4\beta)\epsilon + 2\beta\eta_{\sigma\sigma} + 2(1-\beta)\eta, \end{aligned} \tag{4.6}$$

where α_i in Eq.(4.2) should be replaced by γ_i. We consider $\epsilon \ll 1$ and $\eta_{\sigma\sigma} \ll 1$ and thus

$$n_s \simeq 1 + 2(1-\beta)\eta. \tag{4.7}$$

For $\beta = 0.8$ and $\eta = -0.1$, $n_s \simeq 0.96$.

Since the total primordial power spectrum is not only generated by curvatons in this scenario, some formulations in Sec. 3 should be modified by some powers of β: f_{NL} is replaced by f_{NL}/β^2 and WMAP normalization becomes $P_\zeta = \beta P_{\zeta,WMAP}$. For example, Eq.(3.34) is changed to be

$$f_{NL} < \min\left[10^3 \cdot \beta^{\frac{5}{4}} \cdot c \cdot \left(\frac{r}{N}\right)^{\frac{1}{4}}, \quad 2.3\times 10^3 \cdot \beta^{\frac{4}{3}} \cdot c^{\frac{2}{3}} \cdot \frac{r^{1/6}}{N^{1/3}}\right]. \tag{4.8}$$

For $\beta = 0.8$, the bound on the non-Gaussianity does not change so much and a large value of f_{NL} is still achieved naturally. But we need to stress that a large value of f_{NL} is obtained only when r is not too small.

The size of non-Gaussianity generated by inflaton is controlled by a factor $(1-\beta)^2$ in Eq.(A.18) and rich phenomena are expected in this mixed scenario. A large value of f_{NL}^{inf} is possibly detectable if $(1-\beta)$ is not so small. Another bonus of this mixed scenario is that if the adiabatic fluctuation generated by inflaton is big compared to that from the curvaton ($\beta \sim 0$), our model is relaxed from the constraint on the isocurvature perturbation in [3] even in the case where dark matter was generated before the decay of the curvaton.

5 Random matrix and typical mass spectrum in string theory

Axions are typically present in large numbers in string compactifications, and even when all other moduli are stabilized, the axion potentials remain rather flat as a consequence of well-known nonrenormalization theorems [47]. Following [48] the potential of N axions φ_i is

$$V(\varphi) = \sum_{i=1}^{N} \Lambda_i^4 \left[1 - \cos\left(\frac{\varphi_i}{f_i}\right)\right], \tag{5.1}$$

where f_i is the axion decay constant and Λ_i is the dynamically generated scale of the axion potential that typically arises from an instanton expansion. Redefine the axion field as $\sigma_i \equiv \varphi/f_i$ and then the Lagrangian for small axion displacements $\sigma_i \ll M_p$ in [49] is given by

$$\mathcal{L} = \sum_{i=1}^{N} \left[\frac{1}{2}(\partial\sigma_i)^2 - \frac{1}{2}m_i^2\sigma_i^2\right]. \tag{5.2}$$

In [48,49] the axion fields are taken as inflatons and the value of σ_i is larger than $M_p/\sqrt{N}$, but smaller than M_p. In [35] we argued that the vacuum expectation value of σ_i is bounded by $M_p/\sqrt{N}$ from above and this inflation model might be inconsistent with full quantum theory of gravity. In this section we suggest that these axion fields play the role as curvatons, not inflatons.

It is still difficult to explicitly calculate the mass of axion in the context of KKLT moduli stabilization [50]. However, in [49] the authors found an essentially universal probability distribution for the mass square of axions as the Marcenko-Pastur law

$$p(m^2) = \frac{1}{2\pi v}\frac{\sqrt{(b - m^2/\bar{m}^2)(m^2/\bar{m}^2 - a)}}{m^2}, \tag{5.3}$$

for $a \le m^2/\bar{m}^2 \le b$, where

$$a = (1 - \sqrt{v})^2, \tag{5.4}$$

$$b = (1 + \sqrt{v})^2. \tag{5.5}$$

The shape of the distribution only depends on a single parameter v which is determined by the dimensions of the Kahler and complex structure moduli spaces. This distribution is universal because it does not depend on specific details of the compactification, such as the intersection numbers, the choice of fluxes, or the location in moduli space. It is also insensitive to superpotential corrections. But we cannot determine the overall mass scale from string theory. In a KKLT compactification of Type IIB string theory, there are $h_{1,1}$

axions, and $h_{1,1}+h_{2,1}+1$ is the total dimension of the moduli space (Kahler, complex structure, and dilaton), so that

$$v=\frac{h_{1,1}}{h_{1,1}+h_{2,1}+1}. \tag{5.6}$$

In [49] the authors argued that the models of $v=\frac{1}{2}$ are strongly favored.

In general, the number of axions is roughly $\mathcal{O}(10^2\sim10^3)$ in string theory. So

$$\frac{1}{N}\sum_{i=1}^{N}m_i^{2k}\equiv\langle m^{2k}\rangle\simeq\int_{a\bar{m}^2}^{b\bar{m}^2}m^{2k}p(m^2)dm^2=\bar{m}^{2k}s(k,v) \tag{5.7}$$

up to the order of $1/N$ which can be safely neglected in our analysis, where k is just a number and

$$s(k,v)=\frac{1}{2\pi v}\int_a^b x^{k-1}\sqrt{(b-x)(x-a)}dx. \tag{5.8}$$

The function $s(k,v)$ has some interesting properties:

$$s(0,v)=s(1,v)=s(k,0)=1. \tag{5.9}$$

Since $s(1,v)=1$, $\langle m^2\rangle=\bar{m}^2$ which denotes the overall mass scale. We also define

$$\frac{1}{N}\sum_{i=1}^{N}m_i^{2k}\sigma_i^2\equiv\langle m^{2k}\sigma^2\rangle=\langle m^{2k}\langle\sigma^2\rangle\rangle. \tag{5.10}$$

Here we have $\langle\sigma^2\rangle=\frac{3H_*^4}{8\pi^2m^2}$ and then

$$\sum_{i=1}^{N}m_i^{2k}\sigma_i^2=\frac{3N}{8\pi^2}H_*^4\bar{m}^{2(k-1)}s(k-1,v). \tag{5.11}$$

Because $s(0,v)=1$, $\sum_{i=1}^{N}m_i^2\sigma_i^2=\frac{3NH_*^4}{8\pi^2}$ and Eq.(3.18) is automatically satisfied. Here we assume that $\sigma_i\ll f_i$. Otherwise, the quartic, or higher-power correction terms from the expansion of the axion potential will be important. In string theory, the axion decay constant f_i is generically large [51] and our assumption of $\sigma_i\ll f_i$ is reasonable.

According to Eq.(3.10) and (3.16), we can easily calculate the amplitude of primordial power spectrum and the non-Gaussianity parameter generated by curvatons:

$$P_\zeta^{nc}=6.68\times10^{-6}s(-1/2,v)\frac{NH_*^6}{M_p^4\Gamma_\sigma\bar{m}}, \tag{5.12}$$

$$f_{NL}^{nc}=1.14\times10^{-8}s(-1/4,v)(P_\zeta^{nc})^{-2}\frac{NH_*^8}{M_p^6\Gamma_\sigma^{\frac{3}{2}}\bar{m}^{\frac{1}{2}}}. \tag{5.13}$$

Here are three unknown scales, H_*, Γ_σ and $\bar{m}$, which still cannot be determined by microscopic physics. Canceling Γ_σ, we have

$$f_{NL}^{nc} = 0.66 f_1(v)(P_\zeta^{nc})^{-\frac{1}{2}} N^{-\frac{1}{2}} \frac{\bar{m}}{H_*}, \tag{5.14}$$

where

$$f_1(v) = s(-1/4, v)/s^{\frac{3}{2}}(-1/2, v). \tag{5.15}$$

On the other hand, canceling $\bar{m}$ yields

$$f_{NL}^{nc} = 4.41 \times 10^{-6} f_2(v)(P_\zeta^{nc})^{-\frac{3}{2}} N^{\frac{1}{2}} \frac{H_*^5}{M_p^4 \Gamma_\sigma}, \tag{5.16}$$

with

$$f_2(v) = s(-1/4, v)/s^{\frac{1}{2}}(-1/2, v). \tag{5.17}$$

We have $f_1(1/2) = 0.76$ and $f_2(1/2) = 0.98$. The behaviors of $f_1(v)$ and $f_2(v)$ are showed in Fig. 1. When $v \to 0$, the mass gap of curvatons disappears and we can expect that our

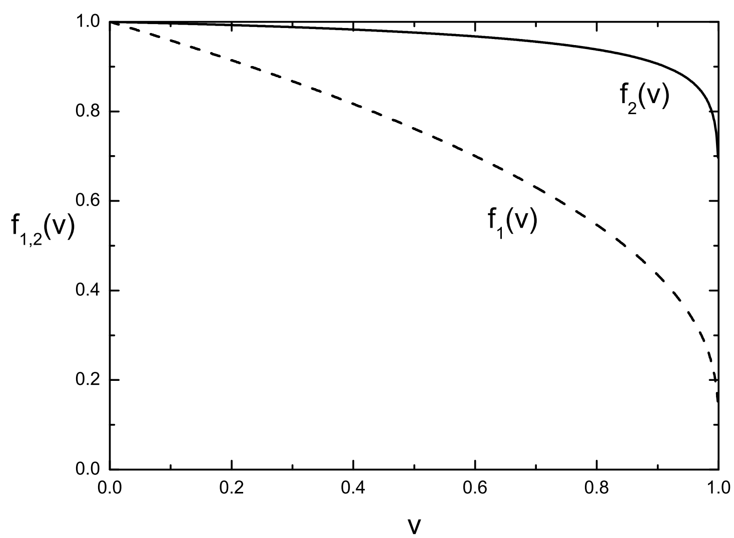

Figure 1: The function $f_1(v)$ and $f_2(v)$.

model is reduced to to the case in Sec. 3.4 where all of curvatons have the same mass. Since $s(k, 0) = 1$ and then $f_1(0) = f_2(0) = 1$, we see both the amplitude of primordial power spectrum and the non-Gaussianity parameter are really the same as those in Sec. 3.4. On the other hand, in Sec. 3.2, we find that the non-Gaussianity parameter f_{NL} is maximized when $m_1 = m_2 = ... = m_N = m$ for keeping H_*, Γ_σ and N fixed. This model is really consistent with our analysis: $f_2(v)$ approaches its maximum value when $v \to 0$.

Here we also know how the mass scale $\bar{m}$ varies with v. For a given f_{NL}^{nc}, $\bar{m} \sim 1/f_1(v)$ rises as v increases.

In this case, we can also calculate $\eta_{\sigma\sigma}$, i.e.

$$\eta_{\sigma\sigma} = \frac{1}{3}\tau(v)\frac{\bar{m}^2}{H_*^2}, \tag{5.18}$$

where

$$\tau(v) = s(1/2, v)/s(-1/2, v). \tag{5.19}$$

The function $\tau(v)$ is illustrated in Fig. 2 and $\tau(1/2) = 0.73$.

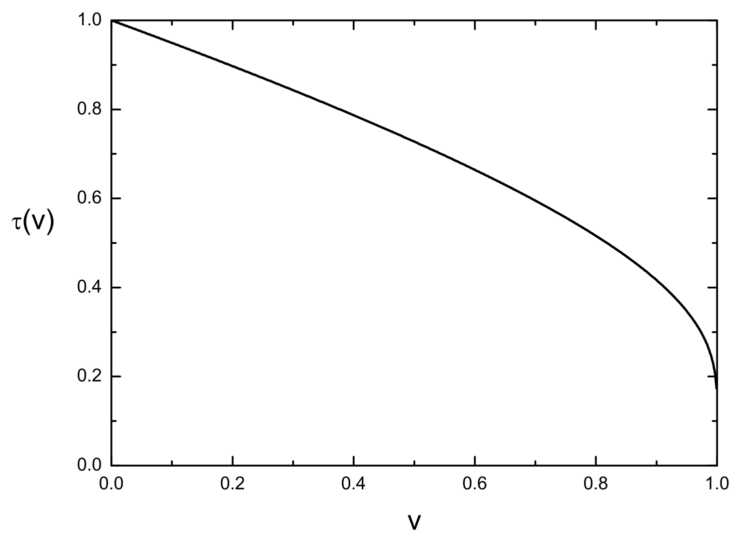

Figure 2: The function $\tau(v)$.

In general, curvatons fluctuations only contribute to a part of the total primordial power spectrum, $P_\zeta^{nc} = \beta P_\zeta^{tot}$, and then $f_{NL} \simeq \beta^2 f_{NL}^{nc}$. Similarly, requiring $\Gamma_\sigma > \bar{m}^3/(c^4 M_p^2)$ yields

$$f_{NL} < \min\left[10^3 \cdot \beta^{\frac{5}{4}} \cdot c \cdot d_1(v) \cdot \left(\frac{r}{N}\right)^{\frac{1}{4}},\ \ 2.3 \times 10^3 \cdot \beta^{\frac{4}{3}} \cdot c^{\frac{2}{3}} \cdot d_2(v) \cdot \frac{r^{1/6}}{N^{1/3}}\right], \tag{5.20}$$

where

$$d_1(v) = s(-1/4, v)/s^{\frac{5}{4}}(-1/2, v), \tag{5.21}$$

$$d_2(v) = s(-1/4, v)/s^{\frac{4}{3}}(-1/2, v). \tag{5.22}$$

The functions $d_1(v)$ and $d_2(v)$ are shown in Fig. 3, and $d_1(1/2) = 0.81$ and $d_2(v) = 0.79$. Again we see that our results are exactly reduced to the case in Sec. 3.4 when the parameter v approaches to zero.

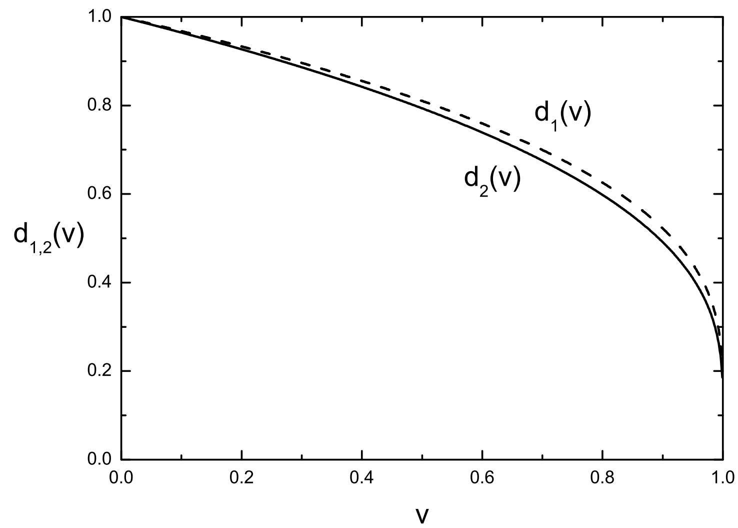

Figure 3: The function $d_1(v)$ and $d_2(v)$.

5.1 Compare to experiments

In this subsection we focus on how to compare our model to experiments. We consider the case with $P_\zeta^{nc} = \beta P_\zeta^{tot}$. There are 8 parameters:

$$\begin{aligned} \text{inflation}: &\quad H_*, \epsilon, \eta \\ \text{N-vaton}: &\quad N, \Gamma_\sigma, \bar{m}, v \\ \text{ratio parameter}: &\quad \beta \end{aligned}$$

Using Eq.(5.18), we have $\frac{\bar{m}}{H_*} = \sqrt{3\eta_{\sigma\sigma}/\tau(v)}$ and then

$$f_{NL} = 1.14\beta^{\frac{3}{2}} f_1(v)(P_\zeta^{tot})^{-\frac{1}{2}} N^{-\frac{1}{2}} \sqrt{\eta_{\sigma\sigma}/\tau(v)}. \tag{5.23}$$

Since $P_\zeta^{inf} = \frac{H_*^2/M_p^2}{8\pi\epsilon} = (1-\beta)P_\zeta^{tot}$, the relationship between tensor-scalar ration r and ϵ is given by

$$r = 16(1-\beta)\epsilon. \tag{5.24}$$

The spectral index is given in Eq.(4.6) as

$$n_s = 1 - (6-4\beta)\epsilon + 2\beta\eta_{\sigma\sigma} + 2(1-\beta)\eta. \tag{5.25}$$

To summarize, there are four quantities which can be measured by experiments: P_ζ^{tot}, $f_{NL}(\beta, v, N, \eta_{\sigma\sigma})$, $r(\beta,\epsilon)$ and $n_s(\beta,\epsilon,\eta,\eta_{\sigma\sigma})$. For a given inflation model, which means ϵ and η are given, the parameters $\eta_{\sigma\sigma}$, N and β can be determined by experiments for the preferable model with $v = 1/2$. Furthermore, if the number of curvatons is given by

the string theory, we can check whether our model is consistent with experiments. For example, Let's consider an inflation model with $\epsilon = 10^{-4}$ and $\eta = -0.1$, and we also assume $\bar{m}/H_* \ll 1$ (or $\eta_{\sigma\sigma} \ll 1$). The tensor perturbation is too small to be detected. For $n_s = 0.96$, $P_\zeta^{tot} = 2.457 \times 10^{-9}$, $f_{NL} = 30$, $v = 1/2$ and $N = 10^3$, we find $\beta = 0.8$ and $\eta_{\sigma\sigma} = 0.0042$. Now $r = 3.2 \times 10^{-4}$, $H_* = 4.36 \times 10^{12}$ GeV and then $\bar{m} = 5.7 \times 10^{11}$ GeV.

6 Discussions

In this paper we explicitly calculate the primordial curvature perturbation in N-vaton model. Multiplicity of light scalar fields is generic in the theories going beyond standard model. Even though the total energy density of these light scalar fields is subdominant during inflation, the perturbation produced by them can dominate the density perturbation on large scale. We also suggest a realistic N-vaton model in which the axions in the KKLT compactification of Type IIB string theory are taken as curvatons, and a rich phenomenology is shown up. If a large local-type non-Gaussianity is confirmed by the forthcoming experiments, it can shed a light on these light scalar fields.

In order to fit the spectral index from WMAP data, the inflaton fluctuation is still required to play a significant role in the total primordial power spectrum. Generally the tensor-scalar ratio is required to be not smaller than 10^{-5} if $f_{NL}^{local} > 10$. Many inflation models constructed in string theory, such as brane inflation [52, 53], happen in a quite low energy scale with $r \sim 10^{-10}$ which is too small to generate a large non-Gaussianity in curvaton/N-vaton scenario. How to construct an inflation model with $r \sim \mathcal{O}(10^{-5} - 10^{-3})$ and $\eta \sim -0.1$ is still an open question.

In general, the curvaton decay rate mediated by particles of mass M_X is expected to be of order m^3/M_X^2. So it is natural to assume that the different curvatons have different decay rates and decay at different time, in particular for the case where they have different masses. The authors in [15] gave a concrete example to show that a large non-Gaussianity can be obtained even when the curvatons dominate the total energy density at the time of decays in the case of two curvatons. However if there are hundreds or thousands of curvatons, whether this enhancement of the non-Gaussianity is generic or not is still an open question. It is worth studying this problem in the future.

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A Appendix: $\zeta_r = \zeta_\phi \neq 0$

This is the most general case. We also expand the curvature perturbation ζ_ϕ to second order as follows

$$\zeta_\phi = \zeta_\phi^{(1)} + \frac{1}{2}\zeta_\phi^{(2)}. \tag{A.1}$$

Now Eq.(2.26) becomes

$$e^{4\zeta} = \left(\sum_{i=1}^{N} \Omega_{\sigma_i} e^{3\zeta_{\sigma_{i,o}}}\right) e^{\zeta} + (1 - \Omega_{\sigma,D}) e^{4\zeta_\phi}. \tag{A.2}$$

Order by order, from the above equation we have

$$\zeta^{(1)} = A \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_{i,o}}^{(1)} + B\zeta_\phi^{(1)}, \tag{A.3}$$

and

$$\begin{aligned} \zeta^{(2)} = & \frac{3A}{2} \sum_{i=1}^{N} \Omega_{\sigma_i,D}(1+h_i) \left(\zeta_{\sigma_{i,o}}^{(1)}\right)^2 - (2 + A\Omega_{\sigma,D}) A^2 \left(\sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_{i,o}}^{(1)}\right)^2 \\ & - 8A^2 B \sum_{i=1}^{N} \Omega_{\sigma_i,D} \zeta_{\sigma_{i,o}}^{(1)} \zeta_\phi^{(1)} + B^2 C \left(\zeta_\phi^{(1)}\right)^2 + B\zeta_\phi^{(2)}, \end{aligned} \tag{A.4}$$

where

$$A = \frac{3}{4 - \Omega_{\sigma,D}}, \quad B = \frac{1 - \Omega_{\sigma,D}}{1 - \Omega_{\sigma,D}/4}, \quad \text{and} \quad C = \frac{3\Omega_{\sigma,D}}{1 - \Omega_{\sigma,D}} A. \tag{A.5}$$

The total curvature perturbation is given by

$$\zeta = \zeta^{(1)} + \frac{1}{2}\zeta^{(2)}. \tag{A.6}$$

We assume all of these fields including curvatons and inflaton are independent. Thus the two different curvatons are uncorrelated with each other

$$\langle \zeta_{\sigma_{i,o}}^{(1)}(\mathbf{k_1}) \zeta_{\sigma_{j,o}}^{(1)}(\mathbf{k_2}) \rangle = (2\pi)^3 \mathcal{P}_{\zeta_{\sigma_{i,o}}}(k_1) \delta_{ij} \delta^3(\mathbf{k_1} + \mathbf{k_2}), \tag{A.7}$$

and curvatons are also decoupled to inflaton ϕ

$$\langle \zeta_{\sigma_{i,o}}^{(1)}(\mathbf{k_1}) \zeta_\phi^{(1)}(\mathbf{k_2}) \rangle = 0. \tag{A.8}$$

The primordial power spectrum $\mathcal{P}_{\zeta_\phi}$ generated by inflaton is defined as

$$\langle \zeta_\phi^{(1)}(\mathbf{k_1}) \zeta_\phi^{(1)}(\mathbf{k_2}) \rangle = (2\pi)^3 \mathcal{P}_{\zeta_\phi} \delta^3(\mathbf{k_1} + \mathbf{k_2}). \tag{A.9}$$

The fluctuations of curvatons and inflaton contribute to the total primordial power spectrum which is given by

$$\mathcal{P}_\zeta^{tot} = \mathcal{P}_\zeta^{nc} + \mathcal{P}_\zeta^{inf}, \tag{A.10}$$

where

$$\mathcal{P}_\zeta^{nc} = A^2 \sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 \mathcal{P}_{\zeta_{\sigma_{i,o}}}, \tag{A.11}$$

is the total curvature perturbation generated by curvatons and

$$\mathcal{P}_\zeta^{inf} = B^2 \mathcal{P}_{\zeta_\phi} \tag{A.12}$$

is the curvature perturbation generated by inflaton. For convenience, we introduce parameters β and γ_i as follows

$$\beta = \mathcal{P}_\zeta^{nc} / \mathcal{P}_\zeta^{tot}, \tag{A.13}$$

$$\mathcal{P}_{\zeta_{\sigma_{i,o}}} = A^{-2} \gamma_i \mathcal{P}_\zeta^{nc}. \tag{A.14}$$

Thus we have

$$\sum_{i=1}^{N} \Omega_{\sigma_i,D}^2 \gamma_i = 1. \tag{A.15}$$

When $\beta = 1$, all of the primordial power spectrum is generated by curvatons and $\gamma_i = \alpha_i$. Now we have

$$\mathcal{P}_{\zeta_{\sigma_{i,o}}} = A^{-2} \beta \gamma_i \mathcal{P}_\zeta^{tot}, \tag{A.16}$$

and

$$\mathcal{P}_{\zeta_\phi} = B^{-2} (1-\beta) \mathcal{P}_\zeta^{tot}. \tag{A.17}$$

Similarly, we can also calculate the total non-Gaussianity parameter f_{NL}^{tot}. Here we only give the result:

$$f_{NL}^{tot} = \beta^2 \tilde{f}_{NL}^{nc} + \beta(1-\beta) \tilde{f}_{NL}^{cross} + (1-\beta)^2 \tilde{f}_{NL}^{inf}, \tag{A.18}$$

where

$$\tilde{f}_{NL}^{nc} = \frac{5}{4A} \sum_{i=1}^{N} \Omega_{\sigma_i,D}^3 \gamma_i^2 (1+h_i) - (\frac{5}{3} + \frac{5A}{6} \Omega_{\sigma,D}), \tag{A.19}$$

$$\tilde{f}_{NL}^{cross} = -\frac{20}{3} A, \tag{A.20}$$

$$\tilde{f}_{NL}^{inf} = \frac{5}{6} C + \frac{1}{B} f_{NL}^{inf}, \tag{A.21}$$

and f_{NL}^{inf} is determined by concrete inflation models [54–63].

B Appendix: Another way to get a large non-Gaussianity in N-vaton model

In this section, we consider that there are two curvatons whose masses are m_1 and m_2 respectively. Without loss of the generality, we assume $m_1 \geq m_2$. Once the Hubble parameter drops below m_1, the curvaton σ_1 starts to oscillate and its energy density goes like $\sim a^{-3}$. Similarly, when $H \sim m_2$, the curvaton σ_2 begins to oscillate. For simplicity, we assume that the universe is dominated by radiation before σ_2 starts to oscillate. The scale factor at the time when σ_1 starts to oscillate is denoted as $a = 1$, and then $a = \sqrt{m_1/m_2}$ when $H \sim m_2$. Since the energy density of an oscillating curvaton goes like $\sim a^{-3}$, the ratio of the energy density between these two curvatons at the time of their decay is

$$x \equiv \frac{\rho_{\sigma_1,D}}{\rho_{\sigma_2,D}} \simeq \sqrt{\frac{m_1}{m_2}} \frac{\sigma_{1,*}^2}{\sigma_{2,*}^2}, \tag{B.1}$$

and then

$$\Omega_{\sigma_1,D} = \frac{x}{1+x}\Omega_{\sigma,D}, \quad \Omega_{\sigma_2,D} = \frac{1}{1+x}\Omega_{\sigma,D}. \tag{B.2}$$

Now we have $\alpha_i = \alpha_c H_*^2/\sigma_{i,*}^2$, where $\alpha_c = A^2/(9\pi^2 P_\zeta)$ is just a numerical coefficient. The constraint on α_i in Eq.(2.36) reads

$$\frac{1}{(1+x)^2}\alpha_c \Omega_{\sigma,D}^2 \frac{H_*^2}{\sigma_{2,*}^2} = \left(1 + \sqrt{\frac{m_1}{m_2}}x\right)^{-1}. \tag{B.3}$$

According to Eq.(2.40), the non-Gaussianity parameter f_{NL} becomes

$$f_{NL} \simeq \frac{5}{4A}\frac{1}{\Omega_{\sigma,D}}\frac{(1+x)(1+\frac{m_1}{m_2}x)}{(1+\sqrt{\frac{m_1}{m_2}}x)^2}. \tag{B.4}$$

If $m_1 = m_2$, f_{NL} is large only when $\Omega_{\sigma,D} \ll 1$. On the other hand, if $m_1 \gg m_2$ and $x \geq \mathcal{O}(1)$, $f_{NL} \sim 1/(x\Omega_{\sigma,D})$ which is large when $\Omega_{\sigma,D} \ll 1$. Now if $x \ll 1$, but $\omega = \sqrt{m_1/m_2}x \geq \mathcal{O}(1)$, we have

$$f_{NL} \sim \frac{5}{4A}\frac{1}{\Omega_{\sigma,D}}\frac{\omega}{(1+\omega)^2}\sqrt{\frac{m_1}{m_2}} \tag{B.5}$$

which can be large even when $\Omega_{\sigma,D} \simeq 1$. However $x \ll 1$ and $\omega \geq \mathcal{O}(1)$ cannot be achieved if the values of curvatons during inflation take the typical values $\sigma_{i,*}^2 = 3H_*^4/(8\pi^2 m_i^2)$, because $x = (m_2/m_1)^{3/2}$ and thus $\omega = x^{2/3} \ll 1$ if $x \ll 1$. Here we also want to remind that the assumption that these two curvatons have the same decay rate is not reasonable if $m_1/m_2 \gg 1$.

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